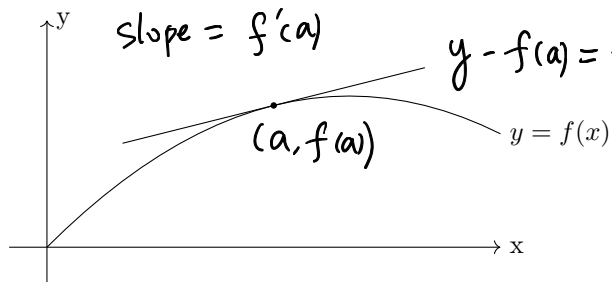


MATH 2D Prep: Tangent lines and Linear Approximation

Facts to Know:

1. Equation and graph of the tangent line to the curve $y = f(x)$ at the point (a, b) :



2. Linear approximation: use the tangent line to approximate $f(x)$ near $(a, f(a))$:

$$f(x) \approx f(a) + f'(a) \cdot (x - a)$$

Examples:

1. Find an equation of the tangent line to the curve $y = e^x$ at the point $(-1, \frac{1}{e})$.

$$f(x) = e^x, \quad \text{slope} = f'(-1) = e^{-1} = \frac{1}{e}$$

$$\text{eqn of tangent line: } y - \frac{1}{e} = \frac{1}{e}(x + 1)$$

2. Use linear approximation to estimate the value of $(2.003)^3$.

$$f(x) = x^3, \quad a = 2, \quad f'(x) = 3x^2, \quad f'(a) = 3 \cdot 2^2 = 12,$$

$$\begin{aligned} (2.003)^3 &= f(2.003) \approx f(2) + f'(2) \cdot (2.003 - 2) \\ &= 8 + 12 \cdot 0.003 = 8.036 \end{aligned}$$

$$\Rightarrow (2.003)^3 \approx 8.036$$